Roll No. | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilators

1. ______________________

2. ______________________

Question Booklet Series **X**

1518

PAPER–II **Question Booklet No.**

(Identical with OMR Answer Sheet Number)

Subject Code : 15

MATHEMATICAL SCIENCES

Time : 2 *Hours* *Maximum Marks:* 200

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D). You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator or mobile phone etc. is strictly prohibited.**
12. **There are no negative marks for incorrect answers.**

[Please Turn Over]

MATHEMATICAL SCIENCES

PAPER II

1. Let $f(x)=\begin{cases} x^2 sin\left(\frac{1}{x}\right)+e^{1/(x-2)}, & x \neq 0,2 \\ 0, & x = 0,2 \end{cases}$

Then f has

(A) removable discontinuity at $x = 0$ and essential discontinuity at $x = 2$

(B) removable discontinuity at $x = 0, 2$

(C) essential discontinuity at $x = 0$ and removable discontinuity at $x = 2$

(D) essential discontinuity at $x = 0, 2$

2. Which of the following satisfies the hypotheses of Rolle's theorem?

(A) $f(x)=|x|$ on [–1, 1]

(B) $f(x)=1-|x-1|$ on [0, 1]

(C) $f(x)=1-(x-1)^{2/3}$ on [0, 2]

(D) $f(x)=x^3-4x$ on [–2, 2]

3. Let $f, g:[a,b]\to[a,b]$ be any two continuous functions $(a,b \in \mathbb{R})$ such that

$$\inf_{x\in[a,b]} f(x) = \inf_{x\in[a,b]} g(x)$$

Then

(A) $f(x)=g(x), \forall x\in[a,b]$

(B) $\exists$ at least one $x_0\in[a,b]$ such that $f(x_0)=g(x_0)$

(C) $\exists$ a unique $x_0\in[a,b]$ such that $f(x_0)=g(x_0)$

(D) $f(x)\neq g(x), \forall x\in[a,b]$

4. Let $f(x,y)=\sqrt{|xy|}, (x,y)\in\mathbb{R}^2$. Then

(A) f is differentiable at the point (0, 0)

(B) f admits directional derivatives along each direction at the point (0, 0)

(C) at least one of the partial derivatives f_x and f_y is non-constant at the point (0, 0)

(D) f is continuous at the point (0, 0)

5. Let $f:\mathbb{C}\to\mathbb{C}$ be an entire function with $f(0) = 1$ and $f(1) = 0$. Then f maps

(A) open sets in $\mathbb{C}$ into open sets in $\mathbb{C}$

(B) closed sets in $\mathbb{C}$ into closed sets in $\mathbb{C}$

(C) open sets in $\mathbb{C}$ into closed sets in $\mathbb{C}$

(D) closed sets in $\mathbb{C}$ into open sets in $\mathbb{C}$

6. Let f be an analytic function in a neighbourhood of '0' having a zero at '0' of order 5 and g has a pole of order 3 at '0'. Then $f(z)/g(z)$ has

(A) zero of order 8 at '0'

(B) pole of order 8 at '0'

(C) pole of order 2 at '0'

(D) zero of order 2 at '0'

7. The function $f(z) = \exp\left(z \big/ (z-1)^2 \right)$ has

(A) no singularity

(B) an essential singularity at z = 1

(C) a double pole at z = 1

(D) a simple pole at z = 1

8. Which of the following functions is analytic?

(A) $f(x+iy) = 6x+5$

(B) $f(x+iy) = \exp(2ix)$

(C) $f(x+iy) = i(x+\sin y)$

(D) $f(x+iy) = 1+2x+2iy$

9. Any group of order 25 is

(A) cyclic

(B) simple

(C) abelian but not simple

(D) abelian and simple

10. Let C[0, 1] denote the ring of all real-valued continuous functions over [0,1] and $\Psi : C[0,1] \to \mathbb{R}$ be the map $\Psi(f) = f(0)$. Then

(A) Ψ is a ring homomorphism onto $\mathbb{R}$ and kernel of Ψ is a maximal ideal of C[0, 1]

(B) Kernel of Ψ is a prime ideal without being a maximal ideal of C[0, 1]

(C) $\Psi(C[0,1]) \subsetneq \mathbb{R}$

(D) Ψ is an isomorphism on C[0, 1] onto $\mathbb{R}$

11. Let $f(x) = x^3 + x^2 + 2$ and $g(x) = x^3 + x + 2$ over $\mathbb{Z}_3$. Then in $\mathbb{Z}_3[x]$

(A) *f* and *g* both are irreducible

(B) *f* is irreduciable but *g* is not irreduciable

(C) *f* is not irreduciable but *g* is irreduciable

(D) none of *f*, *g* is irreducible

12. If *R* is a ring with unity and $a, b \in R$ such that *ab* = 1, then

(A) *ba* is an idempotent in *R* but (1 – *ba*) is not

(B) (1 – *ba*) is an idempotent in *R* but *ba* is not

(C) neither *ba* nor (1 – *ba*) is an idempotent element in *R*

(D) both of *ba* and (1 – *ba*) are idempotent elements in *R*

13. If A is an orthogonal matrix of order 3 with three non-zero eigenvalues $\alpha_1, \alpha_2, \alpha_3$ then the eigenvalues of A^{-1} are

(A) $\alpha_1, \alpha_2, \alpha_3$

(B) $\alpha_1^2, \alpha_2^2, \alpha_3^2$

(C) $\alpha_1, \alpha_2^2, \alpha_3^3$

(D) None of the above

14. Let A and B be real invertible matrices such that AB = – BA. Then

(A) Trace (A) = Trace (B) = 0

(B) Trace (A) = Trace (B) = 1

(C) Trace (A) = 0, Trace (B) = 1

(D) Trace (A) = 1, Trace (B) = 0

15. If two subspaces of $\mathbb{R}^4$ are given by

U = span {(1,2,3,4),(5,7,2,1),(3,1,4,–3)}

V = span {(2,1,2,3),(3,0,1,2),(1,1,5,3)}

then the dimension of U $\cap$ V is

(A) 1

(B) 2

(C) 3

(D) 4

16. Which of the following is false in a metric space (X,d)?

(A) If $AB \subseteq X$ such that A is compact and B is non-compact but closed in X, then $A \cap B$ must be compact

(B) A closed subset of X need not be of second category in X, even if X is complete

(C) Every closed and bounded subset of X is compact in X

(D) Every compact subset of X is closed and bounded in X

17. Let $X_1 = \left(C[0,1], d_1\right)$ and $X_2 = \left(C[0,1], d_2\right)$

where $d_1(f,g) = \sup_{t \in [0,1]} |f(t) - g(t)|$ and

$$d_2(f,g) = \int_0^1 |f(t) - g(t)|\, dt.$$

Then

(A) X_1 and X_2 have different sets of Cauchy sequences

(B) X_1 is complete but X_2 is not

(C) X_2 is complete but X_1 is not

(D) X_1 and X_2 are both complete

18. Let X be a normed linear space and $x_0 \in X$ such that for each bounded linear functional f on X with $\|f\| = 1, \left|f\left(x_0\right)\right| \le k$. Then

(A) $\|x_0\| \le k$

(B) $\|x_0\| = k$

(C) $\|x_0\| > k$

(D) $\|x_0\| = 1$

19. Which of the following is true?

(A) There exists a countably infinite dimensional Banach space

(B) There exists an infinite dimensional normed linear space such that each linear functional on X is continuous

(C) Any infinite dimensional Hilbert space X over $\mathbb{C}$ is isomorphic with l_2

(D) A Hilbert space X is separable iff there exists a complete countable orthonormal set in X

20. Let E be an orthonormal set in a Hilbert space X. Then

(A) $\forall x \in X,\ x = \sum_{y \in E} < x, y > y$

(B) $\forall x, y \in X,\ < x, y >= \sum_{z \in E} < x, z >< \overline{y, z} >$

(C) $\forall x \in X,\ \|x\|^2 = \sum_{z \in E} |< x, z >|^2$

(D) E is complete if the linear hull of E is dense in X.

21. Let f be a linear map from a normed linear space X to a normed linear space Y. Then which of the following is false?

(A) f is continuous on X iff it is continuous at 0

(B) f is necessarily continuous if X is finite dimensional

(C) f is continuous at 0 iff it is uniformly continuous over X

(D) f is always continuous with the choice $X = Y = l_\infty$

22. The Sorgenfrey line is

(A) regular but not normal

(B) separable but not Lindelöf

(C) first countable but not metrizable

(D) connected but not second countable

23. The subspace $Y = Q \times [0,1]$ of $\mathbb{R}^2$ (with Euclidean topology) is

(A) dense in $\mathbb{R}^2$

(B) connected

(C) separable

(D) compact

24. Let X be a topological space such that any two nonempty open sets in X intersect. Then

(A) X is compact

(B) If Y is any other topological space in which any two nonempty open sets intersect, then X is homeomorphic to Y

(C) if $f : X \to \mathbb{R}$ is continuous, then f is bounded but not necessarily constant

(D) every continuous function $f : X \to \mathbb{R}$ is a constant map

25. Let $A=\left\{(x,y)\in\mathbb{R}^2 : y=\sin\frac{1}{x}, 0<x\le 1\right\}$ and $B=\{0\}\times[-1,1]$. Then A ∪ B is

(A) compact but not connected in $\mathbb{R}^2$

(B) locally connected but not compact in $\mathbb{R}^2$

(C) compact and connected in $\mathbb{R}^2$

(D) neither compact nor connected in $\mathbb{R}^2$

26. Any two continuous functions from the real line $\mathbb{R}$ to any discrete space *Y* are

(A) never homotopic

(B) homotopic if one of them is a constant function

(C) homotopic if both are constant functions

(D) always homotopic

27. Union of any two

(A) contractible spaces having non-empty path connected intersertion is contractible

(B) simply connected spaces is contractible

(C) path connected spaces having non-empty path connected intersection is contractible

(D) contractible spaces having non-empty path connected intersection is path connected

28. What is the cardinality of the set $\{Z\in\mathbb{C}/Z^{98}=1$, and $Z^n\neq 1$ for $0<n<98\}$?

(A) 0

(B) 12

(C) 42

(D) 49

29. For a curve on the surface of a cylinder, the curvature (κ) and torsion (τ) scalars are

(A) $\tau=0$, κ can take any real value

(B) $\tau=0$, $\kappa=0$

(C) $\tau\neq 0$, κ can take any real value

(D) $\tau\neq 0, \kappa\neq 0$

30. The differential equation of the geodesics of a space is

(A) a second order linear equation

(B) a second order quasi-linear equation

(C) a first order linear equation

(D) a first order non-linear equation

31. Let $h(x)=\begin{cases}\dfrac{\sin x}{x}, & \text{if } x>0\\ 0, & \text{if } x=0\end{cases}$. Then

(A) h is Lebesgue integrable but not Riemann integrable over $[0,\infty)$

(B) $|h|$ is Lebesgue integrable over $[0,\infty)$

(C) h is Riemann integrable but not Lebesgue integrable over $[0,\infty)$

(D) $|h|$ is Riemann integrable over $[0,\infty)$

32. The contor set k is

(A) disconnected but not totally disconnected

(B) nowhere dense

(C) a closed subset of $\mathbb{R}$ without being perfect

(D) a Borel set with $\mu(k)>0$

33. Suppose a Lebesgue measurable set A in $\mathbb{R}$ contains a non-Lebesgue measurable set, then

(A) $\mu(A)=0$

(B) $\mu(A)>1$

(C) $\mu(A)>0$

(D) A is a Borel set

34. Suppose f and g are two functions from $\mathbb{R}$ to $\mathbb{R}$ with the property: $f(x)=g(x)$ for all irrational points x in $\mathbb{R}$. Then

(A) $f=g$ on the whole of $\mathbb{R}$ if f and g are both Lebesgue measurable on $\mathbb{R}$

(B) $f=g$ on $\mathbb{R}$ if f is continuous and g is Borel measurable on $\mathbb{R}$

(C) $f=g$ on $\mathbb{R}$ if f is continuous on $\mathbb{R}$ and g is Lebesgue measurable on $\mathbb{R}$

(D) $f=g$ on $\mathbb{R}$ if f and g are both continuous on $\mathbb{R}$

35. The root of the equation $f(x)=0$ is found by using the Newton-Raphson method. The initial estimate of the root is $x_0=3$ with $f(3)=5$. The angle the tangent to the function $f(x)$ makes at $x=3$, is 57° with the x axis. The next estimate of the root x_1 most nearly is ($\tan 57°=1{\cdot}54$)

(A) –3·2470

(B) –0·2470

(C) 3·2470

(D) 6·2470

36. Euler's method can be derived by using the first two terms of the Taylor series of y_{i+1} (the value of y at x_{i+1}) in terms of y_i and all the derivatives of y at x_i. If $h = x_{i+1} - x_i$, and the first three terms of the Taylor series are chosen for the ODE, $2\frac{dy}{dx} + 3y = e^{-5x}$, then the explicit expression for y_{i+1} would be

(A) $y_{i+1} = y_i + \frac{1}{2}\left(e^{-5x_i} - 3y_i\right)h$

(B) $y_{i+1} = y_i + \frac{1}{2}\left(e^{-5x_i} - 3y_i\right)h - \left(\frac{5}{2}e^{-5x_i}\right)\frac{h^2}{2}$

(C) $y_{i+1} = y_i + \frac{1}{2}\left(e^{-5x_i} - 3y_i\right)h + \left(-\frac{13}{4}e^{-5x_i} + \frac{9}{4}y_i\right)\frac{h^2}{2}$

(D) $y_{i+1} = y_i + \frac{1}{2}\left(e^{-5x_i} - 3y_i\right)h - 3y_i\frac{h^2}{2}$

37. Let us consider the system of linear equations

$2x + y = 4$

$2x + 1{\cdot}01y = 4{\cdot}02$

Then the condition number is

(A) 2

(B) 602

(C) 501

(D) 500

38. The integral $\int_0^1 \frac{dx}{\sqrt{x}}dx$ can be determined by

(A) Simpson $\frac{1}{3}$ rule

(B) Gaussian quadrature

(C) Trapizoidal rule

(D) Boole's rule

39. For homogeneous deformation defined by

$x_1 = \alpha X_1 + \beta X_2$

$x_2 = -\beta X_1 + \alpha X_2$

$x_3 = \mu X_3$

where $\alpha = \cos\theta$, $\beta = \sin\theta$ (μ, θ, being constants), the Lagrangian strain tensor E has the value as

(A) $\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} \alpha^2+\beta^2-1 & 0 & 0 \\ 0 & \alpha^2+\beta^2-1 & 0 \\ 0 & 0 & \alpha^2+\beta^2-1 \end{bmatrix}$

(C) $\begin{bmatrix} \alpha^2+\beta^2-1 & 0 & -1 \\ 0 & \alpha^2+\beta^2-1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} -\alpha & 0 & 0 \\ 0 & -\beta & 0 \\ 0 & 0 & -1 \end{bmatrix}$

40. Consider the motion $x_i = \left(1+\frac{t}{k}\right)X_i$ where k is a constant and $x_i, X_i (i = 1,2,3)$ are Eulerian and Lagrangian co-ordinates, t is the time. If $\rho = \rho_0$ at $t = 0$, where ρ is the density, then ρ is

(A) $\frac{1}{\left(1-\frac{t}{k}\right)^3}\rho_0$

(B) $\frac{1}{\left(1+\frac{t}{k}\right)^2}\rho_0$

(C) $\frac{1}{\left(1+\frac{t}{k}\right)^3}\rho_0$

(D) $\frac{1}{\left(1-\frac{t}{k}\right)^2}\rho_0$

41. The state of stress at a point is given by

$$(\sigma_{ij}) = \begin{pmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{pmatrix}$$

where a, b, c are constants and σ is some stress. If the stress vector vanishes on a plane normal to $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ then values of a, b, c are

(A) $a = b = c = -\frac{1}{2}$

(B) $a = b = \frac{1}{2}, c = -\frac{1}{2}$

(C) $a = \frac{1}{2}, b = c = -\frac{1}{2}$

(D) $a = b = c = \frac{1}{2}$

42. A particle of mass m and co-ordinate q has the Lagrangian $L = \frac{1}{2}m\dot{q}^2 - \frac{\lambda}{2}q\dot{q}^2$ where λ is a constant. The Hamiltonian for the system is given by

(A) $\frac{p^2}{2(m-\lambda q)}$

(B) $\frac{p^2}{2m} + \frac{\lambda q p^2}{2(m-\lambda q)^2}$

(C) $\frac{p\dot{q}}{2}$

(D) $\frac{p^2}{2m} + \frac{\lambda q p^2}{2m^2}$

43. A stone is dropped from a certain height and is observed to fall the last h cm in t secs. The total time of fall is

(A) $\frac{t}{4}+\frac{h}{gt}$

(B) $\frac{t}{2}+\frac{h}{gt}$

(C) $\frac{t}{4}-\frac{h}{gt}$

(D) $\frac{t}{2}-\frac{h}{gt}$

44. w_{12} is the work done by a conservative field of force $\vec{F}$ from point 1 to point 2 and v_1, v_2 are the potential energies at the point 1 and point 2 respectively, then

(A) $w_{12} = v_1 + v_2$

(B) $w_{12} = v_1 - v_2$

(C) $w_{12} = v_1 v_2$

(D) $w_{12} = v_2 - v_1$

45. Given the Lagrangian L,

$L = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{GM\,m}{r}, \mu = \frac{mM}{m+M}$, then the Routhian R is

(A) $R = -\frac{1}{2}\mu\,\dot{r}^2 - \frac{p_\theta^2}{2\mu\,r^2} - \frac{GM\,m}{r}$

(B) $R = -\frac{1}{2}\mu\,\dot{r}^2 + \frac{p_\theta^2}{2\mu r^2} - \frac{GM\,m}{r}$

(C) $R = -\frac{1}{2}\mu\,\dot{r}^2 + \frac{p_\theta^2}{2\mu r^2} + \frac{GM\,m}{r}$

(D) $R = \frac{1}{2}\mu\,\dot{r}^2 + \frac{p_\theta^2}{2\mu r^2} + \frac{GM\,m}{r}$

46. The curve joining two given points P and Q which is traversed by a particle sliding from P to Q in the shortest time (friction and resistance of the medium are neglected) is

(A) a straight line

(B) a cycloid

(C) a cardioid

(D) a hyperbola

47. The general solution of the system of differential equations $X' = AX$ where $A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$ involving arbitrary constants c_1, c_2 is,

(A) $x = (c_1 + c_2 t)e^{2t}$, $y = -(c_1 + c_2 - c_2 t)e^{2t}$

(B) $x = (c_1 + c_2 t)e^{2t}$, $y = -(c_1 + c_2 + c_2 t)e^{2t}$

(C) $x = (c_1 - c_2 t)e^{2t}$, $y = (c_1 + c_2 - c_2 t)e^{2t}$

(D) $x = (c_1 + c_2 t)e^{2t}$, $y = (c_1 - c_2 + c_2 t)e^{2t}$

48. Boundary condition which includes direct boundary value is

(A) Dirichlet boundary condition

(B) Neumann boundary condition

(C) Forced boundary condition

(D) Discrete boundary condition

49. Let $u(x, t)$ be the solution of IVP, $u_{tt} - u_{xx} = 0$ with $u(x, 0) = x^3$, $u_t(x, 0) = \sin x$. Then $u(\pi, \pi)$ is

(A) $4\pi^3$

(B) π^3

(C) 0

(D) 4

50. The Fourier Transform of 1, where δ is the Dirac-delta function, is

(A) $\frac{1}{\sqrt{2\pi}}\delta$

(B) $\sqrt{2\pi}\,\delta$

(C) $(2\pi)^{\frac{1}{3}}\,\delta$

(D) $2\pi\delta$

51. The solution of the integro differential equation

$y'(x) + e^x \int_0^1 e^{-t}\, y(t)dt = 1,\ y(0) = 0$ is

(A) $y(x) = x + \frac{e-2}{e+2}(1-e^x)$

(B) $y(x) = x - \frac{e-2}{e+2}(1-e^x)$

(C) $y(x) = x + \frac{e+2}{e-2}(1-e^x)$

(D) $y(x) = x - \frac{e+2}{e-2}(1-e^x)$

52. The resolvent kernel of the integral equation $y(x) = 1 + \lambda \int_0^x e^{3(x-t)}\, y(t)\,dt$ is

(A) $e^{(3-\lambda)\,(x-t)}$

(B) $e^{(3+\lambda)\,(x-t)}$

(C) $e^{(3-\lambda)\,(x+t)}$

(D) $e^{(3+\lambda)\,(x-t)}$

53. If a liquid enters into a pipe of diameter d with a velocity v, then its velocity at the exit, when the diameter reduces to 0·5 d, is

(A) v

(B) 0·5 v

(C) 2 v

(D) 4 v

54. Two flows named 1 and 2 are observed. The flow velocities are v_1 and v_2. If all other factors remain the same which flow is more likely to be laminar?

(A) Flow 1 with $v_1 > v_2$

(B) Flow 2 with $v_1 > v_2$

(C) Always flow 1

(D) Always flow 2

55. According to Bernoulli's equation for steady ideal fluid flow,

(A) principle of conservation of mass holds

(B) velocity and pressure are inversely proportional

(C) total energy is constant throughout

(D) the energy is constant along a streamline but may vary across streamlines

56. A transportation problem is a

(A) dual problem

(B) non-linear programming problem

(C) special case of linear programming problem

(D) None of the above

57. Consider an LPP

Maximize $Z = 3x - 2y$

subject to $\frac{x}{\alpha} + \frac{y}{2} = 1$, $x \geq 1$, $y \geq 1$

The first constraint is redundant if

(A) $\alpha < 2$

(B) $\alpha < 3$

(C) $\alpha > 2$

(D) for all α ($\neq 0$)

58. We have to supply customers 100 units of a certain product every Sunday. We collect the product from a local supplier at ₹ 60 per unit. The costs of ordering and transportation from the supplier are ₹ 150 per order. The cost of carrying inventory is estimated at 15% per year of the cost of the product carried.

Then the lot size which will minimize the cost of the system

(A) 316

(B) 416

(C) 532

(D) 281

59. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes. Then the average number of trains in the queue is

(A) 4

(B) 3

(C) 5

(D) 6

60. Let us consider the following optimization problem:

Maximize $Z = f_1(y_1)\ f_2(y_2) \ldots f_n(y_n)$

subject to $a_1 y_1 + a_2 y_2 + \ldots + a_n y_n = b,$

$y_j \geq 0,\ a_j \geq 0$

The recursive functional, to solve this problem by dynamical programming method, is

(A) $F_j(s_j) = \min\limits_{y_j}\left[f_j(y_j) \cdot F_{j-1}(b - s_{j-1})\right],$

$j = n, n-1, \ldots, 2,\ F_1(s_1) = f_1(y_1)$

(B) $F_j(s) = \min\limits_{y_j}\left[f_j(y_j) \cdot F_{j-1}(s_{j-1})\right],$

$j = n, n-1, \ldots, 2,\ F_1(s_1) = f_1(y_1)$

(C) $F_j(s_j) = \max\limits_{y_j}\left[f_j(y_j) \cdot F_{j-1}(b - s_{j-1})\right],$

$j = n, n-1, \ldots, 2,\ F_1(s_1) = f_1(y_1)$

(D) $F_j(s_j) = \max\limits_{y_j}\left[f_j(y_j) \cdot F_{j-1}(s_{j-1})\right],$

$j = n, n-1, \ldots, 2,\ F_1(s_1) = f_1(y_1)$

61. In a two phase simplex method, if in phase-I an artificial variable remains at positive level in the optimal table of phase-I then

(A) the solution is unbounded

(B) there exists no solution

(C) there exists an optimal solution

(D) the problem has multiple solutions

62. In replacement model problems we wish to

(A) maximize operational (maintenance) cost

(B) minimize operational cost

(C) maximize set up cost

(D) minimize set up cost

63. If all the decision variables of an integer programming problem assume 0 or 1, the problem is called

(A) mixed integer programming problem

(B) pure integer programming problem

(C) zero-one programming problem

(D) Both (A) and (C)

64. Which one is incorrect?

(A) Dual of a lattice is not necessarily a lattice

(B) Let $(L, \leq)$ be a lattice and $a, b, c \in L$. If $a \leq b$ and $a \leq c$, then $a \leq b \vee c$

(C) Let $P(A)$ be the power set of any non-empty set A. Then the partially ordered set $(P(A), \leq)$ is a lattice

(D) If $(L, \leq)$ is a lattice with least element 0 and greatest element 1. Then for any $a \in L$, $a \vee 1 = 1$ and $a \wedge 1 = a$

65. Let G be a simple connected planar graph with n vertices and m edges $(m > 2)$, then

(A) $m \leq 3n - 6$

(B) $m \leq 3n + 6$

(C) $m \geq 3n - 6$

(D) $m = 3n - 6$

66. The selection of cricket team is

(A) Random sampling
(B) Systematic sampling
(C) Purposive sampling
(D) Cluster sampling

67. For a population with linear trend, you will prefer

(A) Cluster sampling
(B) Systematic sampling
(C) Stratified sampling
(D) Simple random sampling

68. A simple random sample of size 3 units is drawn with replacement from a population of size N. The probability that the sample consists of three distinct units is

(A) $\frac{1}{N^3}$

(B) $\frac{(N-1)^2}{N^3}$

(C) $\frac{1}{N^2}$

(D) $\frac{(N-1)(N-2)}{N^2}$

69. The ratio of 'between sample variance' to 'within sample variance' follows

(A) F-distribution
(B) χ^2-distribution
(C) Z-distribution
(D) t-distribution

70. Let X_1 and X_2 be two independent N (2, 1) random variables, then the mean and variance of $X_1^2 + X_2^2 - 4X_1 - 4X_2 + 8$ are respectively

(A) 4 and 2
(B) 2 and 4
(C) 4 and 4
(D) 0 and 2

71. Suppose, for i = 1, 2, $X_i | \theta_i \sim N(0, \sigma^2)$ are independently distributed. Under the prior distribution, $\theta_i; i = 1, 2$ are *iid* $N(\mu, \tau^2)$, where σ^2, μ and τ^2 are known. Then, which of the following is true for the marginal distribution of X_1 and X_2 ?

(A) X_1 and X_2 are *iid* $N(\mu, \sigma^2 + \tau^2)$

(B) X_1 and X_2 are not normally distributed

(C) X_1 and X_2 are identical $N(\mu, \sigma^2 + \tau^2)$ but are not independent

(D) X_1 and X_2 are normally distributed but are not identically distributed

72. An insurance company sets up a statistical test with a null hypothesis that the average time for processing a claim is 7 days, and an alternative hypothesis that the average time for processing a claim is greater than 7 days. After completing the statistical test based on 100 clients, it is concluded that the average time is 9 days. However, it is eventually learned that the mean process time is really 7 days. What type of error occurred in the statistical test?

(A) Type-I error

(B) Type-II error

(C) Type-III error

(D) No error occurred in the statistical sense

73. Let $X_1, X_2 ..., X_n$ be a random sample of size n drawn from Bernoulli distribution with parameter $p\,(\frac{1}{2} \le p \le 1)$. The maximum likelihood estimator of p is

(A) $\frac{1}{2}$

(B) $\min\left\{\frac{1}{2}, \overline{X}\right\}$

(C) $\overline{X}$

(D) $\max\left\{\frac{1}{2}, \overline{X}\right\}$

74. Suppose X is a random variable with density function $f(x)$. To test $H_0 : f(x) = 1, 0 < x < 1$ against $H_1 : f(x) = 2x, 0 < x < 1$, the most powerful test of level $\alpha = 0{\cdot}05$

(A) doesn't exist

(B) is to reject H_0 if $x > 0{\cdot}95$

(C) is to reject H_0 if $x > 0{\cdot}05$

(D) is to reject H_0 for $x < C_1$ or $x > C_2$ where C_1 and C_2 have to be determined

75. Fisher's Z-transformation is applied to

(A) sample standard deviation

(B) sample coefficient of variation

(C) sample correlation coefficient

(D) sample proportion

76. The standard chi-squared test for a 2 by 2 contingency table is valid only if

(A) all the expected frequencies are greater than five.

(B) both variables are continuous.

(C) at least one variable is from a normal distribution.

(D) None of the above

77. Binomial distribution tends to Poisson distribution when

(A) $n \to \infty,\ p \to 0$ and $np \to \lambda$ (finite)

(B) $n \to \infty,\ p \to \frac{1}{2}$ and $np \to \lambda$ (finite)

(C) $n \to 0,\ p \to 0$ and $np \to 0$

(D) $n \to k$ (constant), $p \to 0$ and $np \to 0$

78. Critical difference is used in design of experiments to identify the differance between pair of treatments

(A) significantly

(B) insignificantly

(C) indifferently

(D) None of the above

79. Consider 2^5 factorial experiment laid out as a block design with 4 blocks of size 8 each. Suppose, the principal block of this design consists of the treatment combinations (1), ab, de, five others. Which of the following interaction effects can be confounded in this design?

(A) AB, CDE, ABDE

(B) ABC, CDE, ABCDE

(C) AB, BC, AC

(D) AB, CDE, ABCDE

80. Completely Randomised Design (CRD) is

(A) orthogonal design

(B) non-orthogonal design

(C) incomplete design

(D) None of the above

81. The equation of the parabolic trend is $Y = 46 \cdot 6 + 2 \cdot 4X - 1 \cdot 3X^2$. If the origin is shifted backward by three years, the equation of the parabolic trend will be

(A) $Y = 27 \cdot 7 - 5 \cdot 4X - 1 \cdot 3X^2$

(B) $Y = 51 \cdot 1 - 5 \cdot 4X - 1 \cdot 3X^2$

(C) $Y = 27 \cdot 7 + 10 \cdot 2X - 1 \cdot 3X^2$

(D) None of the above

82. Which of the following methods is used to measure seasonal fluctuations in time series data?

(A) Ratio to trend

(B) Ratio to moving average

(C) Link relative

(D) All of the above

83. Harmonic analysis is a method of determining which of the component of a time series?

(A) Trend

(B) Seasonal

(C) Cyclical

(D) Irregular

84. Which of the following is true?

(A) $T_x = \frac{1}{2} l_x + \sum_{t=0}^{\infty} l_{x+t}$

(B) $T_x = \sum_{t=0}^{\infty} l_{x+t}$

(C) $T_x = \frac{1}{2} l_x + \sum_{t=1}^{\infty} l_{x+t}$

(D) None of the above

85. Crude Death Rate (CDR) is considered to be a

(A) ratio measure

(B) Lebesgue measure

(C) growth measure

(D) probability measure

86. Ratio of Net Reproduction Rate (NRR) to Gross Reproduction Rate (GRR) is

(A) less than unity

(B) greater than unity

(C) unity only

(D) None of the above

87. In sequential sampling inspection plan, a lot is rejected if

(A) $\lambda_m > \frac{1-\beta}{\alpha}$

(B) $\lambda_m > \frac{\beta}{1-\alpha}$

(C) $\lambda_m > \frac{\beta}{\alpha}$

(D) $\lambda_m > \frac{1-\beta}{1-\alpha}$

88. In ($\bar{X}$, R) chart, which of the following is true?

(A) Standard values of μ and σ of control limits are known

(B) μ is known but σ is estimated using subgroup quality measures

(C) σ is known but μ is estimated using subgroup quality measures

(D) μ and σ are estimated using subgroup quality measures

89. If lower specification limit is L and upper specification limit is U, the modified control chart is used when

(A) $U - L < 6\sigma$

(B) $U - L = 6\sigma$

(C) $U - L > 6\sigma$

(D) All of the above

90. Consider the linear model

$$y_1 = \theta_1 + 2\theta_2 - 2\theta_3 + \varepsilon_1$$

$$y_2 = \theta_1 + 3\theta_2 - \theta_3 + \varepsilon_2$$

$$y_3 = \theta_2 + \theta_3 + \varepsilon_3,$$

where y_i's are observations, θ_i's are parameters and ε_i's are uncorrelated random variables with mean zero and constant variances. Then which of the following is true?

(A) $2y_1 - y_2 - y_3$ is an unbiased estimator of $\theta_1 - 4\theta_3$

(B) $2y_1 - y_2 + y_3$ is the BLUE of $\theta_1 - 4\theta_3$

(C) $y_2 - 3y_3$ is the BLUE of $\theta_1 - 4\theta_3$

(D) $y_1 - 4y_3$ is an unbiased estimator of $\theta_1 - 4\theta_3$

91. Consider the following linear model

$$y_1 = 2\theta + \beta + e_1$$

$$y_2 = \beta + 2\gamma + e_2$$

$$y_3 = \theta + \beta + \gamma + e_3,$$

where θ, β, γ are unknown parameters and e_1, e_2, e_3 are uncorrelated random errors with zero mean and constant variances. Then which of the following statement is true?

(A) θ, β, γ are estimable

(B) $\theta - \gamma$ is estimable

(C) $4\theta - 2\beta$ is estimable

(D) $\theta + \gamma$ is estimable

92. If $\underset{\sim}{a}'\underset{\sim}{X}$ and $\underset{\sim}{b}'\underset{\sim}{X}$ represent the first and second principal components respectively of a random variable $\underset{\sim}{X}$ following a $N_p(\underset{\sim}{0}, \Sigma)$ distribution, then

(A) $\underset{\sim}{a}'\Sigma\underset{\sim}{a} \geq \underset{\sim}{b}'\Sigma\underset{\sim}{b},\ \underset{\sim}{a}'\underset{\sim}{b} = 0$

(B) $\underset{\sim}{a}'\Sigma\underset{\sim}{a} \leq \underset{\sim}{b}'\Sigma\underset{\sim}{b},\ \underset{\sim}{a}'\underset{\sim}{b} = 0$

(C) $\underset{\sim}{a}'\Sigma\underset{\sim}{a} \geq \underset{\sim}{b}'\Sigma\underset{\sim}{b},\ \underset{\sim}{a}'\Sigma\underset{\sim}{b} = 0$

(D) $\underset{\sim}{a}'\Sigma\underset{\sim}{a} \leq \underset{\sim}{b}'\Sigma\underset{\sim}{b},\ \underset{\sim}{a}'\Sigma\underset{\sim}{b} = 0$

93. Suppose $\begin{pmatrix} \underset{\sim}{x}_{pX1} \\ \underset{\sim}{y}_{qX1} \end{pmatrix}$ follows the $(p+q)$ dimensional normal distribution $N_{p+q}\left(\begin{pmatrix} \underset{\sim}{\mu}_x \\ \underset{\sim}{\mu}_y \end{pmatrix}, \begin{pmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{pmatrix}\right)$, which is non-singular. Then for $\underset{\sim}{x} \in \mathbb{R}^p$, $E(\underset{\sim}{Y} \mid \underset{\sim}{X} = \underset{\sim}{x})$ equals

(A) $\underset{\sim}{\mu}_y + \Sigma_{yx}\Sigma_{xx}^{-1}(\underset{\sim}{x} - \underset{\sim}{\mu}_x)$

(B) $\underset{\sim}{\mu}_y - \Sigma_{yx}\Sigma_{xx}^{-1}(\underset{\sim}{x} - \underset{\sim}{\mu}_x)$

(C) $\underset{\sim}{\mu}_y - \Sigma_{yy}^{-1}\Sigma_{yx}(\underset{\sim}{x} - \underset{\sim}{\mu}_x)$

(D) $\underset{\sim}{\mu}_y + \Sigma_{yy}^{-1}\Sigma_{yx}(\underset{\sim}{x} - \underset{\sim}{\mu}_x)$

94. Let $\underset{\sim}{X}$ be a p-dimensional non-singular multinormal vector with mean vector μ and dispersion matrix $\sum$. Which of the following is not a correct statement?

(A) $(\underset{\sim}{X}-\mu)'\sum^{-1}(X-\mu)$ has chi-square distribution with p degrees of freedom

(B) The maximum likelihood estimators of μ and Σ are unbiased estimators of the parameters

(C) The characteristic roots of Σ are the variances of the principal components of $\underset{\sim}{X}$

(D) Every linear combination of $\underset{\sim}{X}$ is univariate normal

95. A spaceship contains a counter machine that records rays of a certain type of cosmic radiation, that arrive according to a Poisson process at the rate of 10 per hour. If the counter is shut off for 15 minutes at random every hour, then the expected number of rays recorded over a 24 hour period is

(A) 60

(B) 120

(C) 180

(D) 240

96. For a branching chain with offspring generation distribution $\text{Bin}\left(2, \frac{2}{3}\right)$, the probability of extinction is

(A) $\frac{1}{9}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) 1

97. The matrix $\begin{bmatrix} \alpha & \beta & \beta-\alpha \\ \beta & 2\alpha & 2\alpha-\beta \\ \alpha+2\beta & -\alpha & -2\beta \end{bmatrix}$ is a possible Q-matrix for a continuous time Markov chain with 3 states

(A) for no choices of α and β

(B) for all choices of $\alpha \in \left(0, \frac{1}{2}\right)$

(C) for all choices of $\alpha < 0$ and $\beta > 0$

(D) for all choices of $\alpha > 0$ and $\beta < 0$

98. The matrix $\begin{bmatrix} \rho & -\rho \\ -\rho & \rho \end{bmatrix}$ is a valid dispersion matrix of a pair of non-degenerate random variables

(A) for every $\rho \in [0, 1]$

(B) if and only if $\rho = 0$

(C) if and only if $0 < \rho < 1$

(D) if and only if $\rho = 1$

99. If the random variable X takes the values –2, –1, 1 and 2 with respective probabilities $\frac{1}{6}, \frac{1}{6}, \frac{1}{3}$ and $\frac{1}{3}$, then $E\,(X\,|\,X^2)$ equals

(A) 0

(B) $\frac{E\,|\,X\,|}{3}$

(C) $\frac{E(X)}{3}$

(D) $E|X|$

100. If $(X_n)_{n\geq 1}$ is a sequence of independent random variables with X_n taking the values $\frac{1}{n}$ and $-\frac{1}{n}$ with respective probabilites $\frac{3}{4}$ and $\frac{1}{4}$ for every $n \geq 1$, then the sequence

$\frac{1}{n^{\alpha}}(X_1 + 2X_2 + ... + nX_n)$ converges in probability to a non-zero random variable

(A) for no choice of α

(B) if and only if $\alpha = \frac{1}{2}$

(C) if and only if $\alpha = 1$

(D) if and only if $\alpha = 2$

ROUGH WORK

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